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Universe generation from black hole interiors

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Abstract: We point out that scenarios in which the universe is born from the interior of a black hole may not posses many of the problems of the Standard Big-Bang (SBB) model. In particular we demonstrate that the horizon problem, flatness, and the structure formation problem might be solved naturally, not necessarily requiring a long period of cosmological inflation. The black hole information loss problem is also discussed. Our conclusions are completely independent of the details of general models.

Keywords: Black Holes in String Theory, Models of Quantum Gravity, Cosmology of Theories beyond the SM, Physics of the Early Universe.

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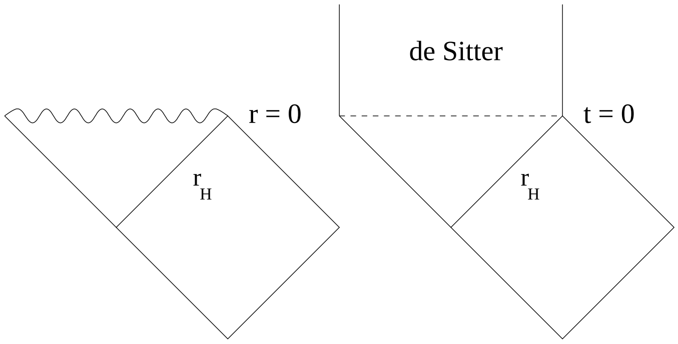
Contents

1. Introduction

In this note we consider the properties of a daughter universe spawned from the interior of a black hole resting in a mother universe. We show that such a daughter universe will take on certain properties of the mother universe. Possible solutions to the horizon, flatness, structure formation and black hole information loss problems are suggested. Many cosmological scenarios in which our universe emerges from the interior of a black hole have been proposed (see e.g. [1]–[8]). Here we provide motivation for studying such scenarios by briefly reviewing some of the relevant work.

In [1, 2], it was postulated without any constructive realization (but based on the limiting curvature conjecture) that in the high Weyl curvature region near the Schwarzschild singularity a matching between the Schwarzschild and the de Sitter metrics as depicted in figure 1 takes place. A constructive implementation of the limiting curvature hypothesis was discovered in [3]. In this construction, the gravitational action involves higher derivative terms parameterized by means of a non-dynamical scalar field ϕ with a potential $V(\phi)$. The higher derivative terms are found using the "Limiting Curvature Construction", demanding that at high curvatures the metric become locally de Sitter [3]. Based on this principle, it was possible to construct a non-singular black hole in $1+1$ dimensions [4]. This scenario closely resembles the one we have in mind in this paper.[1]

[1] We are currently extending the construction of [3] to discuss the four-dimensional Schwarzschild black hole.



(a) (b)

Figure 1: Figure (a) shows a Penrose diagram of a Schwarzschild black hole with event horizon r_{H} and singularity at $r = 0$. Figure (b) shows a non-singular black hole with de Sitter interior.

In the scenario of [5], a study of non-critical string cosmology in two spatial dimensions reveals a non-singular black hole. An observer crossing the black hole's horizon enters into a new universe. A related phenomenon occurs in the Pre-Big-Bang (PBB) [6] model which is based on the low energy effective action of string theory. In this scenario the universe starts out in the string perturbative vacuum state, in a sea of dilaton and gravitational waves. Quantum fluctuations lead to the formation of black holes in the Einstein frame which correspond to dilaton driven inflationary phases in the string frame. At late times, the universe resulting from this construction evolves in accordance with the Standard Big-Bang model.

In [7] the authors analyze cosmology within the framework of the AdS/CFT correspondence. They consider the motion of a charged domain wall that separates an external Reissner-Nordstrom region of spacetime from an interior de Sitter region. In this context, solutions exist such that a black hole forms with an interior resembling a toy cosmological model.

Finally, in [8] a model is constructed where a black hole forms within the scope of a classical field theory with false vacuum interior and true vacuum exterior. Although the metric on the outside of the black hole is Schwarzschild, the interior metric is de Sitter. The action is the Einstein-Hilbert action modified by the inclusion of a scalar field and a potential for that scalar field.

Similar ideas were also put forward by Smolin [9]. He postulates that the universe may have been born out of the interior of a black hole which in turn is embedded in a parent universe. Characteristics of the parent universe are passed on to the daughter universe. This is a cosmological analogy to Darwin's "survival of the fittest" theory of natural selection because only universes with conditions suitable to form black holes are capable of reproducing. Although Smolin provides no quantitative realization of his model, the arguments of [9] are nonetheless intriguing and address many of the problems of the SBB.

Due to the extensive work in this area it is clear that the idea that our universe might be generated from the inside of a black hole is of great interest, and that several scenarios have been formulated which produce this exact result. In this paper we will discuss a number of features that all such scenarios have in common. Note that the conclusions described in this paper are *independent* of the specific models.

The basic set up we have in mind is represented in the Penrose diagrams of figure 1. As is evident from this diagram, when an observer crosses over the event horizon, r and t switch roles (the Schwarzschild metric components g_{tt} and g_{rr} change sign). In (a), the Schwarzschild black hole has a singularity at $r = 0$. In (b), the singularity is replaced by an initial time surface of the de Sitter universe. In other words, the location $r = 0$ from the viewpoint of an observer in the mother universe becomes the initial time surface $t = 0$ for an observer in the baby de Sitter universe. Hence all of the matter that will fall into the black hole, eventually reaching the same place ($r = 0$) but at different times will enter into the baby universe at the same time ($t = 0$) but in different places.

2. Solving problems of SBB cosmology

Many of the problems of the SBB[2] can be solved if the universe is created inside a black hole which is resting in a parent universe. In this section we will suggest how the horizon, flatness and structure formation problems might be solved without requiring a long period of inflation. It is interesting to note however, that all of the scenarios in [1]–[8] do involve a de Sitter bounce (which is presumably of too short duration to solve the problems of standard cosmology).

One of the major problems of the SBB model which is solved by inflation is the horizon problem. The horizon problem is essentially a problem of causality. The region over which the CMB is observed to be homogeneous to better than one part in 10^4 is much larger than the comoving forward light cone of an observer at the time of recombination, which is the maximal distance over which micro-physical forces could have caused the homogeneity.

Inflationary models solve this problem. During inflation the forward light cone expands exponentially. If inflation lasts for a sufficient number of e-foldings, the forward light cone will be larger than the past light cone at the time of last scattering.

Inflation is not without its own problems. In particular, inflation is not able to address the fluctuation problem, super-Planck-scale physics problem, initial singularity problem and the cosmological constant problem as described in [10]. It is therefore desirable to search for possible alternative solutions to the problems of the SBB model.

[2]For a recent review on the problems of the SBB and inflationary models see [10].

JHEP06(2001)024

2.1 The horizon problem

Now we will describe how the causality aspect of the horizon problem is solved naturally within the context of models which are of the type depicted in figure 1. First of all, note that the forward light cone of a point on the horizon at $t = 0$ (where t is the Schwarzschild time coordinate) encompasses all points along the radial direction in the de Sitter universe. All that needs to be shown therefore is that causal contact is possible along the angular directions. Thus, if it can be shown that a particle travels sufficiently far (about 180°) around the black hole, then it follows that there can be causal contact between all points of the new universe at the time of the bounce. It is then probable that particles which fall into the black hole will interact frequently before reaching $r = 0$, establish thermal equilibrium and thus homogenize. Hence, when matter enters the new universe it will already be isotropic.

Here we will examine the geodesic equations for Schwarzschild geometry in order to follow the trajectory of a particle in orbit around a black hole. We will then search for orbits which exhibit the desired behavior.

The Schwarzschild metric is

$$ds^2 = -\left(1 - \frac{2m}{r}\right) dt^2 + \frac{dr^2}{(1 - \frac{2m}{r})} + r^2 d\Omega^2 \,, \tag{2.1}$$

where $d\Omega^2$ is the metric on the unit sphere. The equations of motion are

$$\begin{aligned}
\ddot{t} + \frac{2m}{r^2}\frac{1}{(1-\frac{2m}{r})}\dot{t}\dot{r} &= 0 \,, \\
\ddot{r} + \frac{m}{r^2}\left(1-\frac{2m}{r}\right)\dot{t}^2 - r\sin^2\theta\left(1-\frac{2m}{r}\right)\dot{\phi}^2 - & \\
- r\left(1-\frac{2m}{r}\right)\dot{\theta}^2 - \frac{m}{r^2}\frac{1}{(1-\frac{2m}{r})}\dot{r}^2 &= 0 \,, \\
\ddot{\theta} - \cos\theta\sin\theta\dot{\phi}^2 + \frac{2}{r}\dot{\theta}\dot{r} &= 0 \,, \\
\ddot{\phi} + 2\cot\theta\dot{\phi}\dot{\theta} + \frac{2}{r}\dot{\phi}\dot{r} &= 0 \,,
\end{aligned} \tag{2.2}$$

where the dot indicates differentiation with respect to proper time τ.

It is now possible to derive the equation for the orbit of a particle moving in a Schwarzschild background, $\phi(r)$. For simplicity we will take the particle's trajectory to lie in the equatorial plane, $\theta = \pi/2$, $\dot{\theta} = \ddot{\theta} = 0$. The EOM (2.2) simplify and are easily solved to give

$$\phi(r) = \int \frac{L\, dr}{r^2\sqrt{E^2 - \left(1-\frac{2m}{r}\right)\left(\frac{L^2}{r^2}+\kappa\right)}} \,, \tag{2.3}$$

where $\kappa = 1$ for matter and $\kappa = 0$ for photons. Here we have defined constants for the energy $E = (1 - 2m/r)\dot{t}$ and angular momentum $L = r^2\dot{\phi}$.[3] Upon making the substitution $u = 1/r$ eq. (2.3) becomes

$$\phi(u) = -\int \frac{du}{\sqrt{2mu^3 - u^2 + \frac{2m\kappa}{L^2}u - \frac{\kappa}{L^2} + \frac{E^2}{L^2}}} . \tag{2.4}$$

For a rigorous treatment of the above integral see [11]. The geometries of the geodesics are determined by the roots of the function beneath the square root. For our purposes it is sufficient to make a few approximations. We will break down the analysis by studying three relevant regions around the black hole. First we will consider the behavior of orbits at large r. Next, we consider orbits of particles with initial position barely inside the event horizon (i.e. with $r \approx 2m$ and $r < 2m$). Finally we will consider orbits deep inside the horizon close to the singularity.

2.1.1 Small u approximation

In the small u (large r) regime the initial particle position is far from the black hole and the u^3 term in (2.4) can be ignored. Upon integration the equation of the orbit becomes

$$\phi(u) = \arcsin \frac{2(\frac{m\kappa}{L^2} - u)}{\sqrt{-\Delta}} , \tag{2.5}$$

where $\Delta = -4(E^2 - \kappa + \frac{\kappa^2 m^2}{L^2})/L^2$. For simplicity we will consider photon trajectories and take $\kappa = 0$. Hence,

$$\phi(u) = \arcsin\left(\frac{-Lu}{E}\right) . \tag{2.6}$$

This equation will admit orbits of the desired type. We could have guessed this behavior since in the large-r regime the orbits are appropriately described by Newtonian gravity. Bound orbits which circle the black hole numerous times before falling into the singularity will have ample time to interact with near horizon matter and reach thermal equilibrium before plunging to $r = 0$.

2.1.2 Near-horizon approximation

Consider orbits of photons starting *just* inside the event horizon. The least significant term in this regime is the u^2 term. The integral (2.4), with $\kappa = 0$ reduces to

$$\phi(u) = -\int \frac{du}{\sqrt{2mu^3 + \frac{E^2}{L^2}}} . \tag{2.7}$$

We start the photon at a distance of $r_i \approx 1.9m$ and let it fall all the way to the singularity, $r_f = 0$. We use the same values for E, m and L as [11].[4] The results are

[3] The E and L are constants since the lagrangian associated with the metric (2.1) is not explicitly dependent on t and ϕ.

[4] Note that varying these values does not seem to change the results significantly.

JHEP06(2001)024

that the photon travels through an angle of $\approx 114°$ before falling into the singularity. Hence, some photons will be able to reach thermal equilibrium. Probably not enough, however, to solve the horizon problem. As one would expect, the result turns out to be even less promising when we start near the singularity.

2.1.3 Large u approximation

For very small values of r, far inside the horizon the dominant term in (2.4) is the u^3 term. Keeping only this term and integrating we find

$$\phi(\Delta r) = \frac{2}{\sqrt{2m}}(\Delta r)^{1/2}\,, \tag{2.8}$$

where $\Delta r = r_f - r_i$. Here r_i is the initial radial value of the particle (with $r < 2m$) and r_f is the final radius which we take to be extremely close to the singularity.

From (2.8), we deduce that orbits of this type are unable to "bend" enough to traverse an angle ϕ anywhere near the desired 180°. For example, if we take $r_i = m$ (near the largest r where our approximation is still valid) and let this particle fall to the singularity at $r_f = 0$ we find a maximum angle of $\phi \approx 80°$. It appears that in order for the black hole mechanism described above to solve the horizon problem we will need to rely on matter reaching equilibrium before it crosses the horizon. Such geodesics are discussed in section 2.1.1.

Hence we have successfully addressed the horizon problem. Firstly, the causality aspect of the horizon problem disappears because there is causal contact between all points in space at the beginning of the de Sitter phase. In addition, matter contained in a homogeneous parent universe will necessarily produce a homogeneous daughter universe via the black hole mechanism discussed above. Even if the parent universe is not homogeneous, the matter falling into the black hole may have ample time to reach equilibrium by following geodesics of the type described in section 2.1.1. Matter which has not come into causal contact by the time it has reached the horizon is less likely to achieve equilibrium before entering the de Sitter region (see 2.1.2 and 2.1.3). Thus, it appears that one cannot solve the horizon problem by considering interactions which occur solely inside the horizon.

2.2 The flatness problem

Another problem of the SBB is the flatness problem. Observations indicate that a sizeable fraction of the critical matter density is contained in galaxy clusters and therefore Ω, the parameter defined to be the ratio of the density of matter in the universe measured today ρ_0 to the critical density ρ_c needed to give a spatially flat universe, is of the order of $\Omega = 1$. Assuming an initial spectrum of adiabatic scale-invariant density fluctuations, the recent observations of the cosmic microwave background (CMB) anisotropies [12, 13] indicate that Ω is very close to 1, a value which in standard cosmology is an unstable fixed point in an expanding universe.

JHEP06(2001)024

If the universe were not nearly flat, it is very likely that we would not exist. If Ω is larger than 1 the universe would have collapsed long ago, perhaps even on the order of the Planck time, 10^{-43} seconds. If Ω is less than 1 the universe would have expanded too fast to allow for structures such as galaxies to form. As flatness is an unusual condition among the class of standard models one might expect cosmology to reveal a mechanism which requires flatness (such as inflation).

It is possible to reformulate the flatness problem in terms of the parameter k in the Friedmann-Robertson-Walker line element of relativistic cosmology,

$$ds^2 = -dt^2 + R^2(t)\left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right). \tag{2.9}$$

Here k gives the geometry of 3-spaces of constant curvature and can be -1, 0, 1. $R(t)$ is the scale factor. If $k = -1$ or 0, the universe is open and if $k = 1$ the universe is closed and will collapse. It is possible to calculate the energy density of the universe at the present instant ρ_0 in terms of k by using the Einstein equations and the metric (2.9),

$$\rho_0 = \frac{3}{8\pi G}\left(\frac{k}{R_0^2} + H_0^2\right). \tag{2.10}$$

The flatness problem now translates into explaining why $k = 0$, since $\rho_c = 3H_0^2/8\pi G$.

The argument we use to solve the flatness problem (i.e. explain why $k = 0$) is very simple. In order to smoothly connect the black hole exterior to the de Sitter interior the spatial topology of both regions must be the same. Therefore, if the parent black hole has one topology, so must the daughter universe, generated from the mechanism discussed above. In our model we are cutting out the innermost region of a black hole manifold and matching it to half of de Sitter space. Whereas the space-time topology of de Sitter space is fixed, the topology of the spatial sections depends on the space-time slicing selected by the physics (see e.g. the textbook [14] for a detailed discussion of this issue). We will now argue that in our construction we must match the inside of a black hole manifold to the flat spatial sections of de Sitter space (given by the coordinates $\hat{t}, \hat{x}, \hat{y}, \hat{z}$ in [14]).

The full space-time diagram of the model is sketched in figure 2. In this scenario, the spatial size of the daughter universe is given by the length of the dotted line. If the daughter universe is infinite in spatial extent the parameter k must be either -1 or 0. This length is easily calculated using the Schwarzschild metric. Note, that as one crosses the event horizon space and time change roles. With the same inevitability that an observer outside of the black hole must move forward in time, grow old and die an observer inside the horizon can only move forward in "space" eventually reaching the center of the black hole. It is therefore sensical to make the variable substitution $r \to -t$ in the Schwarzschild metric, and the interior metric

JHEP06(2001)024

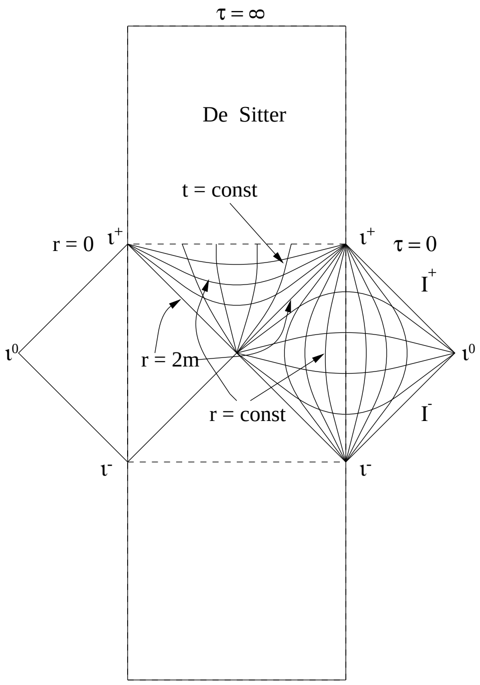


Figure 2: The space-time diagram of a non-singular Schwarzschild black hole with de Sitter interior. The length of the dotted line gives the spatial size of the universe. The location of this line is $r = 0$ in the Schwarzschild coordinates and corresponds to the initial time surface $\tau = 0$ of the de Sitter regime. Note that the roles of r and t switch when crossing the horizon.

(with $\theta = 0$ and $\phi = \text{const}$) becomes

$$ds^2 = \left(1 + \frac{2m}{t}\right) dr^2 - \frac{dt^2}{\left(1 + \frac{2m}{t}\right)} \,. \tag{2.11}$$

Note that the horizon is now located at $t = -2m$. To calculate the size of the universe we may integrate along a line of constant t near $t = 0$ over all space (see figure 2):

$$s = \int_{-\infty}^{\infty} \sqrt{\left(1 + \frac{2m}{t}\right)} dr \,, \tag{2.12}$$

which is clearly infinite.

It is possible to match the above result with a calculation performed from the de Sitter perspective. For a locally de Sitter universe, the scale factor for the metric (2.9) grows like,

$$\begin{aligned} & \sinh H\tau \qquad (k = -1) \\ R \propto\ & \cosh H\tau \qquad (k = 1) \\ & \exp H\tau \qquad (k = 0) \,. \end{aligned} \tag{2.13}$$

Here we have introduced the variable τ to represent the time coordinate of the de Sitter Universe. Thus, the size of the spatial sections of the de Sitter universe is given by integrating ds along a surface of constant τ (see figure 2):

$$s = \int_{-\infty}^{\infty} \frac{R(\tau)\, dr}{\sqrt{1 - kr^2}}\,. \tag{2.14}$$

Note that the integral (2.12) tends to infinity linearly. The only value for k in (2.14) which causes the integral to linearly approach infinity is the $k = 0$, flat solution. Therefore, in order to match the spatial topology of the de Sitter universe to that of the black hole we must be matching to the spatially flat slices of de Sitter space, i.e. have $k = 0$.[5]

2.3 The structure formation problem

It is generally assumed that structures in the universe were formed by gravitational instabilities from some initial perturbations. Explaining the origin of these perturbations is one of the main challenges for modern cosmology, a challenge which inflationary cosmology quite successfully meets: quantum fluctuations produced during the de Sitter phase on sub-Hubble scales are the seeds for the observed perturbations (see e.g. [15] for a comprehensive review).

In the scenario discussed above in which our universe emerges as the inside of a black hole via a short de Sitter bounce, a quantitative theory of the origin and evolution of fluctuations remains to be worked out. Here, we mention some ideas.

First, it is possible that cosmic string defects generated during a matter phase transition in the new universe provide the seeds for structure (see e.g. [16, 17, 18] for comprehensive reviews). At the moment, it appears that such a mechanism has problems explaining the observed [12, 13] narrow first "Doppler" peak in the spectrum of CMB anisotropies.

It may be possible that Hawking radiation of the parent black hole could provide a solution to the structure formation problem in the new universe. The radiation appears as quantum fluctuations in the de Sitter space. If the wavelength of a fluctuation becomes greater than the particle horizon of the de Sitter space ($l \gg c/H$) causality forces the fluctuation to become frozen as a classical amplitude that can seed structure formation.

2.4 The black hole information loss problem

Yet another problem which is naturally solved in this scenario is the black hole information loss problem. Consider a large star of mass M which is initially described by a pure quantum state $|\psi\rangle$ having zero entropy. Now imagine this star undergoes gravitational collapse and forms a black hole. Before the collapse the density matrix

[5] The issue of global topology has not yet been addressed.

of the star is given by $\rho = |\psi\rangle\langle\psi|$. After collapse, the black hole begins to decay via thermal Hawking radiation and is described by a mixed state, with corresponding density matrix

$$\rho = \sum_n p_n |\psi_n\rangle\langle\psi_n| \ . \tag{2.15}$$

Here the p_n are the probabilities of having different initial states. Since the entropy of the Hawking radiation is of order $M^2/m_{\rm pl}^2$ where $m_{\rm pl}$ is the Planck mass, the final state is missing information as compared to the initial state. If the black hole decays via radiation until it completely disappears it will take any information about the matter which has fallen into it with it. This leads to a fundamental contradiction with the laws of quantum mechanics which state that a pure state cannot evolve into a mixed state. In the scenario depicted above all pure states which fall into the black hole emerge in the new universe as pure states and the information loss problem is avoided.

3. Discussion and conclusions

In this paper we have studied consequences of cosmological scenarios in which our universe is born from a black hole resting in a parent universe [1]–[8]. We have discussed the ways in which some of the problems of the SBB model may be solved given such a model. In particular, we have provided a solution to the horizon problem by examining geodesics of matter falling into a black hole and showing that it is possible to bring this matter into causal contact before it emerges in the new universe.

A possible solution to the flatness problem was discussed. The size of the daughter universe was calculated from both the Schwarzschild and the de Sitter perspectives. The Schwarzschild calculation predicts the universe to be infinite. The integral representing the length approaches infinity linearly. In order to recover the same result in the de Sitter frame, we must match the interior of the black hole to the spatially flat sections of de Sitter space. This is the only way to make the length approach infinity linearly. Hence, it appears that our model singles out the observed, flat FRW universe via a topological argument.

We showed that this scenario does not suffer from the black hole information loss problem since pure states evolve to pure states and information is transferred from the parent universe to the black hole interior universe. Finally, a relation between structure formation and Hawking radiation was suggested.

There are a number of interesting problems which will be left for future research. One is to construct a higher derivative theory of gravity which will provide a realization of the above mentioned model. The new gravitational action should admit a solution which resembles the Schwarzschild black hole at large distances but with the singularity replaced by a de Sitter universe. The construction will be similar to that in [19]. It will be interesting to consider applications of the AdS/CFT correspon-

JHEP06(2001)024

dence to such models. We would also like to compare thermodynamical quantities of the exterior black hole to those of the de Sitter universe inside. The effects of black hole evaporation on the universe may prove interesting.

Acknowledgments

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JHEP06(2001)024